Confirmatory Factor Analysis I

Introduction to Structural Equation Modeling

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Outline

General Ideas

Latent Variables

A *latent variable* is simply an unobservable variable

- Usually something we want to analyze
- The mathematical formalization of a *Hypothetical Construct*

Latent variables are very common.

- A patient's current mood
- The approval rate of a political leader
- The health of an ecosystem
- The number of ”calories” burned by an hour of cardio training
- The useful lifespan of an industrial machine

Factor Analysis

Factor analysis (FA) is a statistical method for estimating latent variables.

- We estimate a set of latent variables that parsimoniously explain the relations among a pool of observed variables.
- Factor analyses can be exploratory (*Exploratory Factor Analysis*, EFA) or confirmatory (*Confirmatory Factor Analysis*, CFA).

In FA, we are primarily interested in estimating the underlying *measurement model*.

- The latent structure through which a set of latent variables generate a set of observed items.
- Hence, principal components analysis (PCA) is not true FA.

Latent Factors

A *latent factor* or *latent construct* is the specific type of latent variable that we're estimating in FA.

- To estimate a latent factor, we need multiple, substantively overlapping *indicator variables*.
- All latent factors are latent variables, but not all latent variables are latent factors.

Latent factors are very common in social and behavior science because we often use multi-item questionnaire to measure the various unobservable aspects of society and human experience.

- Mental health screeners (e.g., Beck Depression Inventory, Child Behavior Checklist)
- Intelligence tests (e.g., WAIS, WISC)
- Customer satisfaction surveys

Indicator Variables

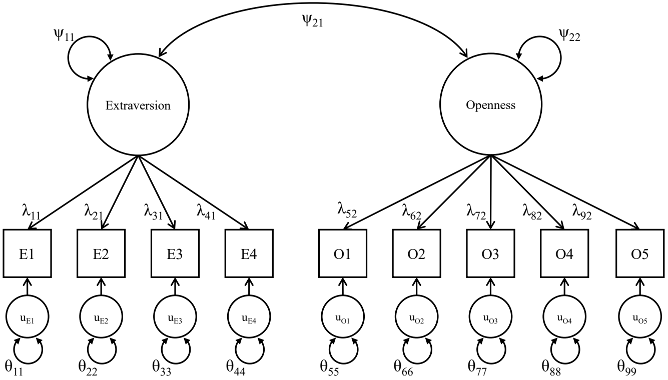
The *indicator variables* are the observed variables that we use to estimate a latent factor.

- The latent factor is never observed, so we need to infer its characteristics from the information provided by our observed data.
- A *measurement model* is a one-to-many mapping of latent factors onto indicator variables.

Indicator variables go by many names

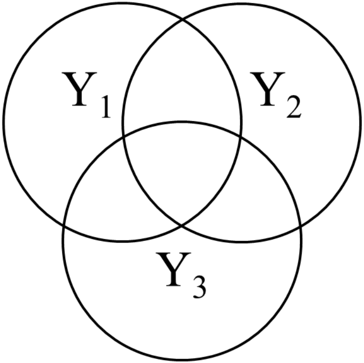
- Indicators
- Observed variables
- Observed items
- Pretty much any sensible combination of the words: *observed*, *variable*, *item*, *indicator*

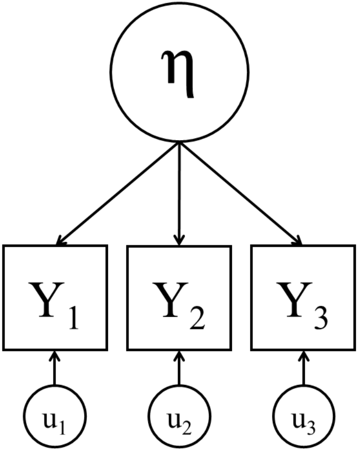
Example CFA



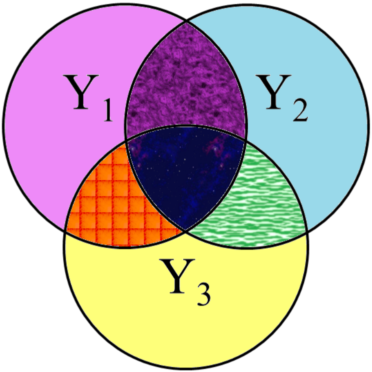
ψ21
ψ11
ψ22
Extraversion
Openness
λ11
λ21
λ31
λ41
λ52
λ62
λ72
λ82
λ92
E1
E2
E3
E4
O1
O2
O3
O4
O5
uE1
uE2
uE3
uE4
uO1
uO2
uO3
uO4
uO5
θ11
θ22
θ33
θ44
θ55
θ66
θ77
θ88
θ99

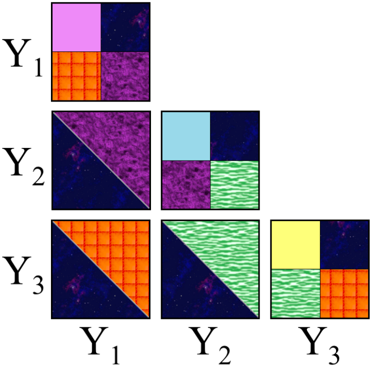
CFA Intuition



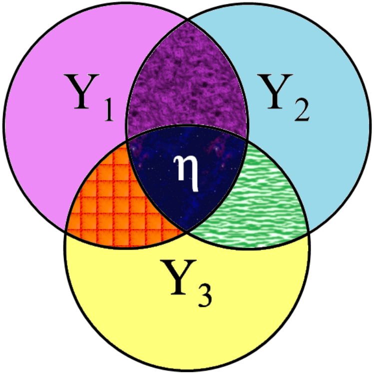


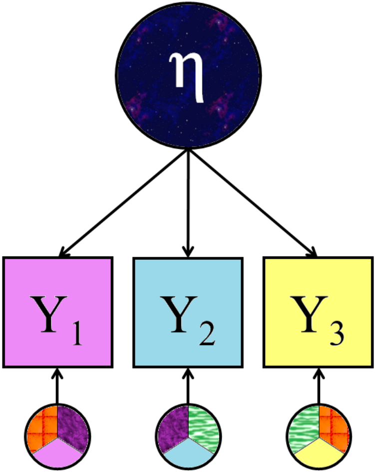
Partitioning Variance





CFA as Partitioned Variance





DIFFERENT TYPES OF FACTOR ANALYSIS

Two Subscales of Extraversion

Having Fun

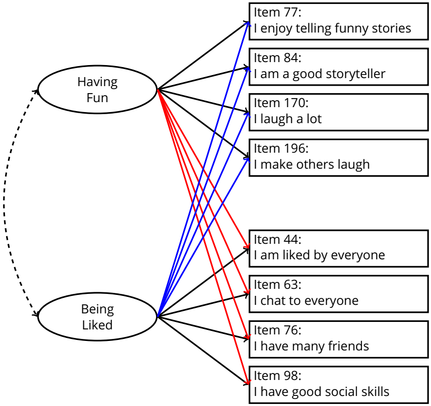
- Item 77: I enjoy telling funny stories
- Item 84: I am a good storyteller
- Item 170: I laugh a lot
- Item 196: I make others laugh

Being Liked

- Item 44: I am liked by everyone
- Item 63: I chat to everyone
- Item 76: I have many friends
- Item 98: I have good social skills

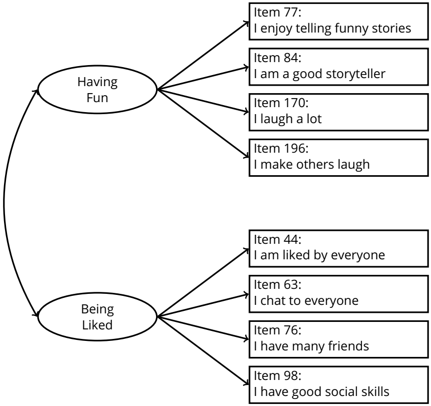
Exploratory Factor Analysis

- All items load onto all factors
- No hypothesized measurement model
- Estimating latent covariances is optional
  - Oblique factors → Estimated
  - Orthogonal factors → Fixed
- Solution is not unique
- Use rotation to improve interpretability

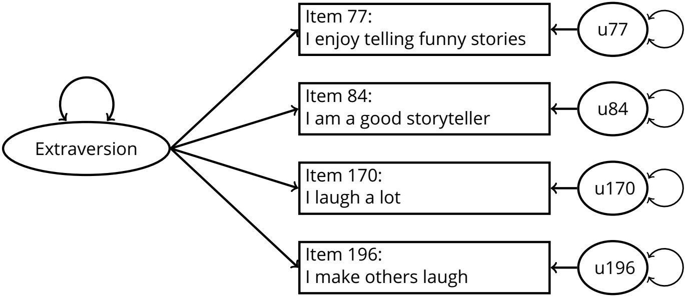


Confirmatory Factor Analysis

- The statistical model represents the hypothesized measurement model
- No cross-loadings unless they're predicted by theory
- Almost always estimate the latent covariances
- A unique solution exists



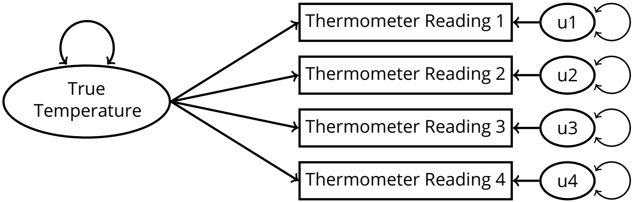
Reflective Constructs



In a reflective measurement model, the items are the dependent variables, and the latent factor is the independent variable.

- The observed items are dependent variables.
- The latent factor is causing the items to take the values we observe.

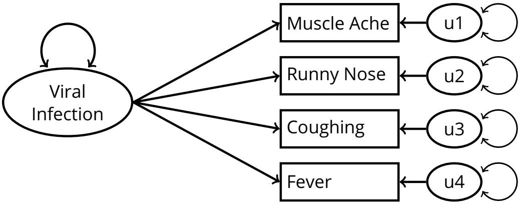
Reflective Constructs



The true temperature is the underlying (unobserved) factor that produces thermometer readings.

- Any given thermometer reading is only an imperfect reflection of the true temperature.
- Multiple readings increase the reliability of our temperature estimate.

Reflective Constructs



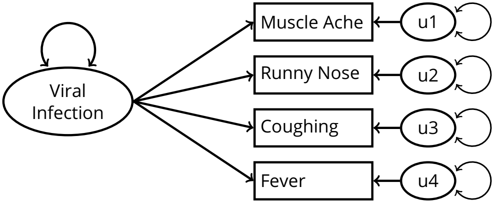
The latent viral infection is the causal factor that gives rise to the observed symptoms of illness.

- Symptoms are the dependent variables.
- Viral infection is the unobserved predictor variable.

Reflective Constructs

For reflective constructs, the underlying measurement model is a multivariate linear regression.

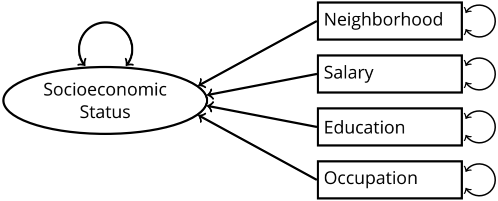
- The latent construct is the X variable.
- The observed indicators are the Y variables.
- The model reproduces the indicators via a system of linear equations.



$$
\begin{aligned}
Y_{ache} &= \lambda_{11} VI + u_1 \\
Y_{nose} &= \lambda_{21} VI + u_2 \\
Y_{cough} &= \lambda_{31} VI + u_3 \\
Y_{fever} &= \lambda_{41} VI + u_4
\end{aligned}
$$

Formative Constructs

Flipping the direction of the factor loadings makes a *formative construct*.



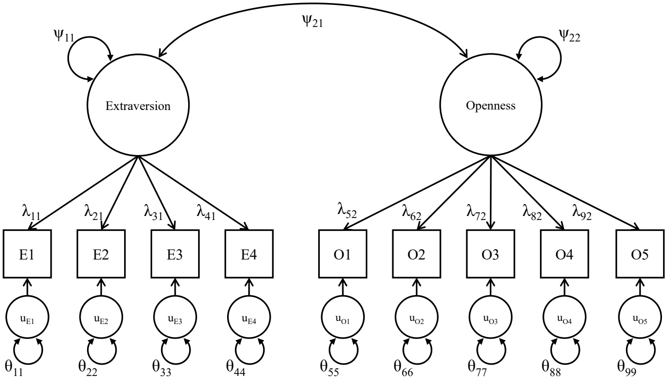
SES is an *index* defined as a (weighted) sum of the observed items.

- SES is the (latent) dependent variable, predicted by the items.
- This model is not empirically testable.

Technical Details

Diagram: Covariance Structure Model

The basic CFA only models the covariance structure.



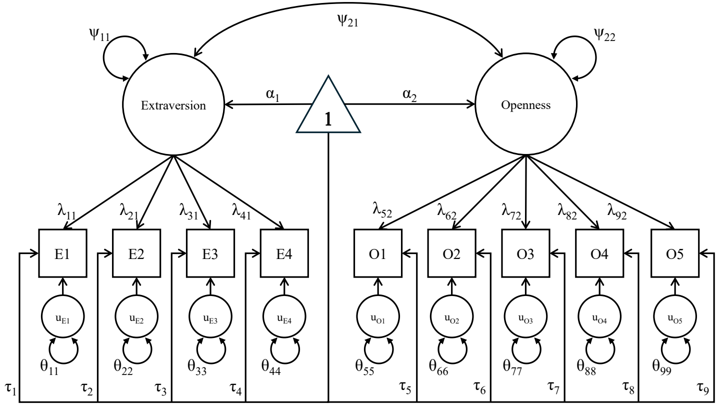
Matrices: Covariance Structure Model

$$\Psi = \begin{bmatrix} \psi_{11} & \\ \psi_{21} & \psi_{22} \end{bmatrix}$$

$$\Theta = \begin{bmatrix} \theta_{11} & & & & & & & & \\ 0 & \theta_{22} & & & & & & & \\ 0 & 0 & \theta_{33} & & & & & & \\ 0 & 0 & 0 & \theta_{44} & & & & & \\ 0 & 0 & 0 & 0 & \theta_{55} & & & & \\ 0 & 0 & 0 & 0 & 0 & \theta_{66} & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & \theta_{77} & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_{88} & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_{99} \end{bmatrix} \qquad \Lambda = \begin{bmatrix} \lambda_{11} & 0 \\ \lambda_{21} & 0 \\ \lambda_{31} & 0 \\ \lambda_{41} & 0 \\ 0 & \lambda_{52} \\ 0 & \lambda_{62} \\ 0 & \lambda_{72} \\ 0 & \lambda_{82} \\ 0 & \lambda_{92} \end{bmatrix}$$

Diagram: Mean & Covariance Structure

We can model the mean structure by including the latent means and item intercepts in our model.



Matrices: Mean & Covariance Structure

$$\alpha = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \quad \Psi = \begin{bmatrix} \psi_{11} & \\ \psi_{21} & \psi_{22} \end{bmatrix}$$

$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \\ \tau_5 \\ \tau_6 \\ \tau_7 \\ \tau_8 \\ \tau_9 \end{bmatrix} \quad \Theta = \begin{bmatrix} \theta_{11} & & & & & & & & \\ 0 & \theta_{22} & & & & & & & \\ 0 & 0 & \theta_{33} & & & & & & \\ 0 & 0 & 0 & \theta_{44} & & & & & \\ 0 & 0 & 0 & 0 & \theta_{55} & & & & \\ 0 & 0 & 0 & 0 & 0 & \theta_{66} & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & \theta_{77} & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_{88} & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_{99} \end{bmatrix} \quad \Lambda = \begin{bmatrix} \lambda_{11} & 0 \\ \lambda_{21} & 0 \\ \lambda_{31} & 0 \\ \lambda_{41} & 0 \\ 0 & \lambda_{52} \\ 0 & \lambda_{62} \\ 0 & \lambda_{72} \\ 0 & \lambda_{82} \\ 0 & \lambda_{92} \end{bmatrix}$$

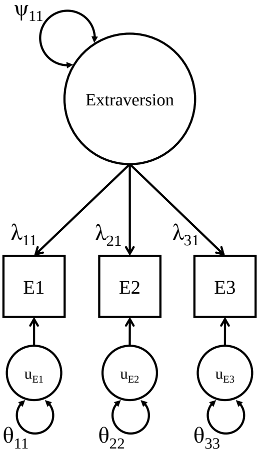
Working Example

To keep things clear, let's start with a simpler example.

$$\Psi = \begin{bmatrix} \psi_{11} \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} \lambda_{11} \\ \lambda_{21} \\ \lambda_{31} \end{bmatrix}$$

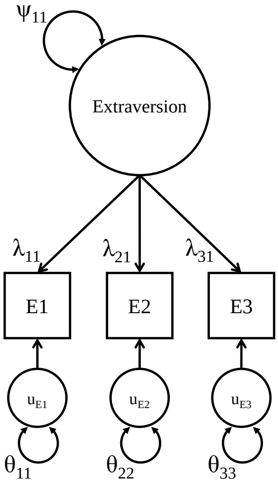
$$\Theta = \begin{bmatrix} \theta_{11} & & \\ 0 & \theta_{22} & \\ 0 & 0 & \theta_{33} \end{bmatrix}$$



Number of Estimated Parameters

For this example, we need to estimate seven parameters to fully define the measurement model.

- One latent variance: ψ_{11}
- Three factor loadings: $\{\lambda_{11}, \lambda_{21}, \lambda_{31}\}$
- Three residual variances: $\{\theta_{11}, \theta_{22}, \theta_{33}\}$



Available Information

The data only contain a fixed amount of information.

- We can quantify the available information in discrete units.
- Every unique element of $\text{Cov}(\mathbf{Y})$ contributes one unit of information.

$$\text{Cov}(\mathbf{Y}) = \begin{bmatrix} var(y_1) & & \\ cov(y_2, y_1) & var(y_2) & \\ cov(y_3, y_1) & cov(y_3, y_2) & var(y_3) \end{bmatrix}$$

In this example, we have six pieces of available information.

- Three variances: $var(y_1)$, $var(y_2)$, $var(y_3)$
- Three covariances: $cov(y_2, y_1)$, $cov(y_3, y_1)$, $cov(y_3, y_2)$

For a positive-definite $M \times M$ covariance matrix, the number of unique elements will always be $Q = \frac{M(M+1)}{2}$.

Degrees of Freedom

We can only estimate one parameter for each piece of available information, Q.

- We can estimate no more than Q parameters in any one model.

The *degrees of freedom* (df) is the difference between the amount of information available in the data, Q, and the number of parameters estimated in our model, P.

$$df = Q - P$$

If $df < 0$, the model is not estimable.

- A model with $df < 0$ is *not identified*.
- The data do not provide enough information to define a unique solution for all P parameter estimates.

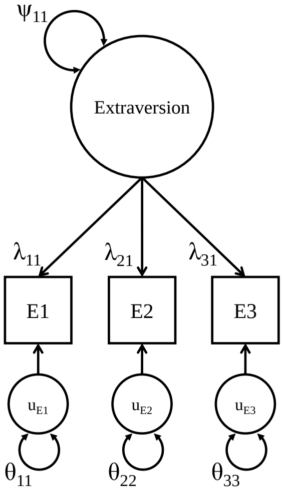
Degrees of Freedom

What are the degrees of freedom for our example?

$$Q = \frac{3(3+1)}{2} = 6$$
$$df = Q - P = 6 - 7 = -1$$

This model has negative df.

- We cannot estimate the model in this form.
- We must impose *identifying constraints*.



Identifying Constraints

Consider the following equation:

$$5 = x + y$$

What are the values of x and y?

Identifying Constraints

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Identifying Constraints

Consider the following equation:

$$5 = x + y$$

What are the values of x and y?

$$y = 5 - x$$

What if we assume that $y = x$?

$$5 = x + y$$
$$0 = x - y$$

Identifying Constraints

Consider the following equation:

$$5 = x + y$$

What are the values of x and y?

$$y = 5 - x$$

What if we assume that $y = x$?

$$5 = x + y$$
$$0 = x - y$$

Now we have enough information:

$$5 = x + x = 2x \quad \Rightarrow \quad x = y = 2.5$$

Identifying Constraints

We must fix some parameters to identify the model.

- For each construct, we need $df \geq 0$.
- If the construct has three or more indicators:
  - Fix one parameter in the covariance model.
  - Fix one parameter in the mean model.
- If the construct has two indicators:
  - Fix an additional parameter in the covariance model.
- If the construct has only one indicator:
  - Cannot define a latent factor.

Identifying Constraints

These constraints also define the scale of the latent factors.

- Latent factors have no direct representation as observed variables in our dataset.
- A latent factor only exists after we've estimated it.
- So latent factors have no inherent scale.
- Identifying constraints are also called *scaling constraints*.

There are two common methods of identifying/scaling CFA models.

1. Marker-variable method
2. Fixed-factor method

Unconstrained Model

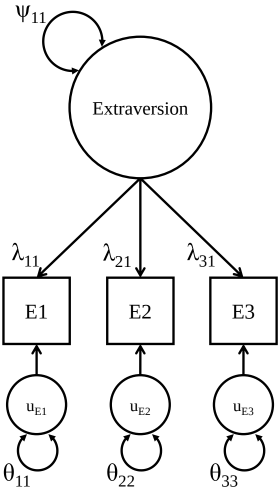
$$\Psi = \begin{bmatrix} \psi_{11} \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} \lambda_{11} \\ \lambda_{21} \\ \lambda_{31} \end{bmatrix}$$

$$\Theta = \begin{bmatrix} \theta_{11} & & \\ 0 & \theta_{22} & \\ 0 & 0 & \theta_{33} \end{bmatrix}$$

$$Q = \frac{3(3+1)}{2} = 6$$

$$df = Q - P$$
$$= 6 - 7 = -1$$



Marker-Variable Method

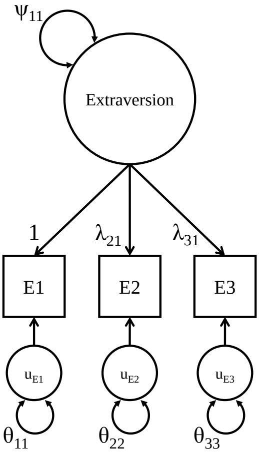
Constrain the covariance model by fixing one factor loading to 1.

$$\Psi = \begin{bmatrix} \psi_{11} \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} 1 \\ \lambda_{21} \\ \lambda_{31} \end{bmatrix}$$

$$\Theta = \begin{bmatrix} \theta_{11} & & \\ 0 & \theta_{22} & \\ 0 & 0 & \theta_{33} \end{bmatrix}$$

$$\begin{aligned} df &= Q - P \\ &= 6 - 6 = 0 \end{aligned}$$



Fixed-Factor Method

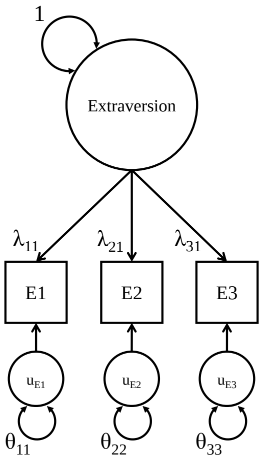
Constrain the covariance model by fixing one factor variance to 1.

$$\Psi = \begin{bmatrix} 1 \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} \lambda_{11} \\ \lambda_{21} \\ \lambda_{31} \end{bmatrix}$$

$$\Theta = \begin{bmatrix} \theta_{11} & & \\ 0 & \theta_{22} & \\ 0 & 0 & \theta_{33} \end{bmatrix}$$

$$\begin{aligned} df &= Q - P \\ &= 6 - 6 = 0 \end{aligned}$$



Unconstrained Model with Means

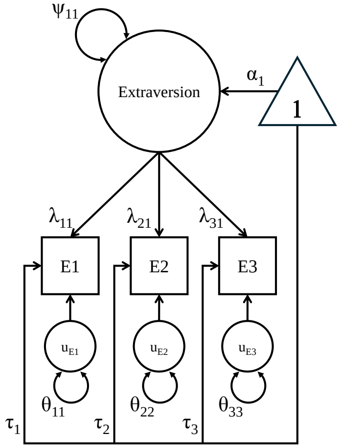
$$\Psi = \begin{bmatrix}\psi_{11}\end{bmatrix} \quad \alpha = \begin{bmatrix}\alpha_1\end{bmatrix}$$

$$\Lambda = \begin{bmatrix}\lambda_{11} \\ \lambda_{21} \\ \lambda_{31}\end{bmatrix} \quad \tau = \begin{bmatrix}\tau_1 \\ \tau_2 \\ \tau_3\end{bmatrix}$$

$$\Theta = \begin{bmatrix}\theta_{11} & & \\ 0 & \theta_{22} & \\ 0 & 0 & \theta_{33}\end{bmatrix}$$

$$Q = \frac{3(3+1)}{2} + 3 = 9$$

$$df = Q - P$$
$$= 9 - 11 = -2$$



Marker-Variable with Means

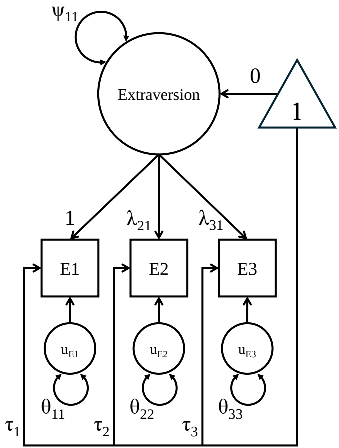
Constrain the mean model by fixing the latent mean to zero.

$$\Psi = \begin{bmatrix}\psi_{11}\end{bmatrix} \quad \alpha = \begin{bmatrix}0\end{bmatrix}$$

$$\Lambda = \begin{bmatrix}1 \\ \lambda_{21} \\ \lambda_{31}\end{bmatrix} \quad \tau = \begin{bmatrix}\tau_1 \\ \tau_2 \\ \tau_3\end{bmatrix}$$

$$\Theta = \begin{bmatrix}\theta_{11} & & \\ 0 & \theta_{22} & \\ 0 & 0 & \theta_{33}\end{bmatrix}$$

$$\begin{aligned} df &= Q - P \\ &= 9 - 9 = 0 \end{aligned}$$



Marker-Variable with Means

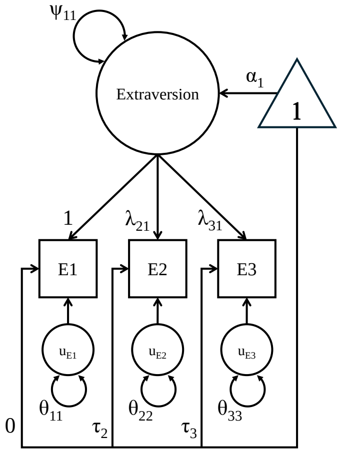
Constrain the mean model by fixing one item intercept to zero.

$$\Psi = \begin{bmatrix} \psi_{11} \end{bmatrix} \quad \alpha = \begin{bmatrix} \alpha_1 \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} 1 \\ \lambda_{21} \\ \lambda_{31} \end{bmatrix} \quad \tau = \begin{bmatrix} 0 \\ \tau_2 \\ \tau_3 \end{bmatrix}$$

$$\Theta = \begin{bmatrix} \theta_{11} & & \\ 0 & \theta_{22} & \\ 0 & 0 & \theta_{33} \end{bmatrix}$$

$$\begin{aligned} df &= Q - P \\ &= 9 - 9 = 0 \end{aligned}$$



Fixed-Factor with Means

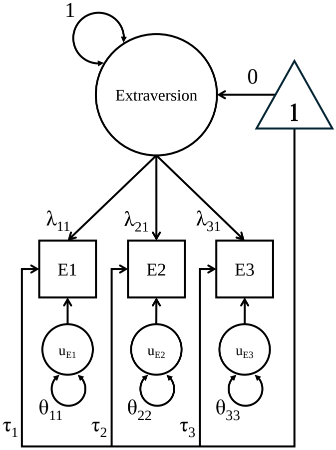
Constrain the mean model by fixing the latent mean to zero.

$$\Psi = \begin{bmatrix} 1 \end{bmatrix} \quad \alpha = \begin{bmatrix} 0 \end{bmatrix}$$

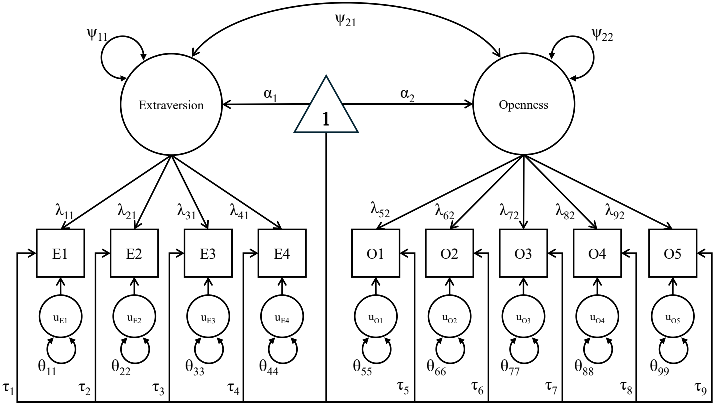
$$\Lambda = \begin{bmatrix} \lambda_{11} \\ \lambda_{21} \\ \lambda_{31} \end{bmatrix} \quad \tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix}$$

$$\Theta = \begin{bmatrix} \theta_{11} & & \\ 0 & \theta_{22} & \\ 0 & 0 & \theta_{33} \end{bmatrix}$$

$$\begin{aligned} df &= Q - P \\ &= 9 - 9 = 0 \end{aligned}$$



Two Construct: Unconstrained Model

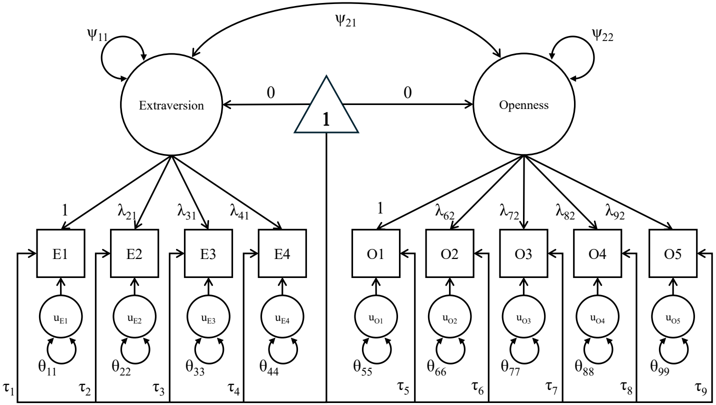


Unconstrained Parameter Matrices

$$\alpha = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \quad \Psi = \begin{bmatrix} \psi_{11} & \\ \psi_{21} & \psi_{22} \end{bmatrix}$$

$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \\ \tau_5 \\ \tau_6 \\ \tau_7 \\ \tau_8 \\ \tau_9 \end{bmatrix} \quad \Theta = \begin{bmatrix} \theta_{11} & & & & & & & & \\ 0 & \theta_{22} & & & & & & & \\ 0 & 0 & \theta_{33} & & & & & & \\ 0 & 0 & 0 & \theta_{44} & & & & & \\ 0 & 0 & 0 & 0 & \theta_{55} & & & & \\ 0 & 0 & 0 & 0 & 0 & \theta_{66} & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & \theta_{77} & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_{88} & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_{99} \end{bmatrix} \quad \Lambda = \begin{bmatrix} \lambda_{11} & 0 \\ \lambda_{21} & 0 \\ \lambda_{31} & 0 \\ \lambda_{41} & 0 \\ 0 & \lambda_{52} \\ 0 & \lambda_{62} \\ 0 & \lambda_{72} \\ 0 & \lambda_{82} \\ 0 & \lambda_{92} \end{bmatrix}$$

Two Construct: Marker Variable

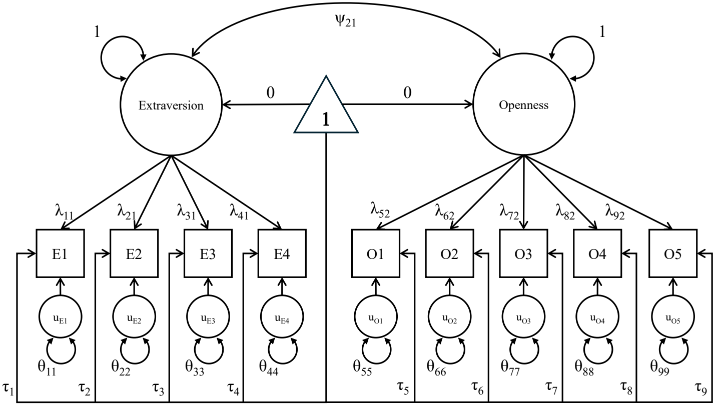


Parameter Matrices: Marker Variable

$$\alpha = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \Psi = \begin{bmatrix} \psi_{11} & \\ \psi_{21} & \psi_{22} \end{bmatrix}$$

$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \\ \tau_5 \\ \tau_6 \\ \tau_7 \\ \tau_8 \\ \tau_9 \end{bmatrix} \quad \Theta = \begin{bmatrix} \theta_{11} & & & & & & & & \\ 0 & \theta_{22} & & & & & & & \\ 0 & 0 & \theta_{33} & & & & & & \\ 0 & 0 & 0 & \theta_{44} & & & & & \\ 0 & 0 & 0 & 0 & \theta_{55} & & & & \\ 0 & 0 & 0 & 0 & 0 & \theta_{66} & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & \theta_{77} & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_{88} & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_{99} \end{bmatrix} \quad \Lambda = \begin{bmatrix} 1 & 0 \\ \lambda_{21} & 0 \\ \lambda_{31} & 0 \\ \lambda_{41} & 0 \\ 0 & 1 \\ 0 & \lambda_{62} \\ 0 & \lambda_{72} \\ 0 & \lambda_{82} \\ 0 & \lambda_{92} \end{bmatrix}$$

Two Construct: Fixed-Factor



Parameter Matrices: Fixed-Factor

$$\alpha = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \Psi = \begin{bmatrix} 1 & \\ \psi_{21} & 1 \end{bmatrix}$$

$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \\ \tau_5 \\ \tau_6 \\ \tau_7 \\ \tau_8 \\ \tau_9 \end{bmatrix} \quad \Theta = \begin{bmatrix} \theta_{11} & & & & & & & & \\ 0 & \theta_{22} & & & & & & & \\ 0 & 0 & \theta_{33} & & & & & & \\ 0 & 0 & 0 & \theta_{44} & & & & & \\ 0 & 0 & 0 & 0 & \theta_{55} & & & & \\ 0 & 0 & 0 & 0 & 0 & \theta_{66} & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & \theta_{77} & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_{88} & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_{99} \end{bmatrix} \quad \Lambda = \begin{bmatrix} \lambda_{11} & 0 \\ \lambda_{21} & 0 \\ \lambda_{31} & 0 \\ \lambda_{41} & 0 \\ 0 & \lambda_{52} \\ 0 & \lambda_{62} \\ 0 & \lambda_{72} \\ 0 & \lambda_{82} \\ 0 & \lambda_{92} \end{bmatrix}$$

EXAMPLE

South African Personality Inventory Project

SAPI©

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Nel, J. A., Valchev, V. H., Rothmann, S., van de Vijver, F. J. R., Meiring, D., & de Bruin, G. P. (2012). Exploring the personality structure in the 11 languages of South Africa. *Journal of Personality*, *80*, 915–948.

SAPI details

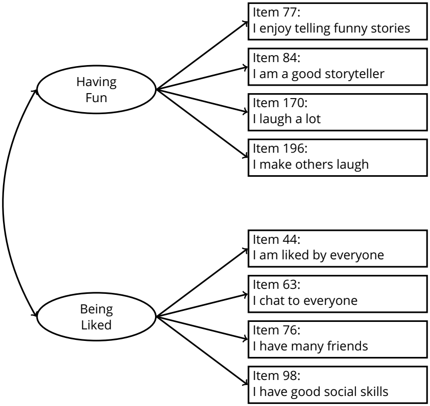
- 1216 participants from 11 official language groups
- From about 50,000 descriptive responses to 262 personality items
- Nine personality clusters:
  - Conscientiousness
  - Emotional Stability
  - Extraversion
  - Facilitating
  - Integrity
  - Intellect
  - Openness
  - Relationship Harmony
  - Soft-Heartedness (Ubuntu)
- Our data: selection of 1000 participants

CFA of Extraversion Items

Suppose we hypothesize two distinct dimensions of extraversion underlying 8 of the SAPI items.

1. Having Fun
2. Being Liked by Others

We'll define our measurement model as the two-factor CFA shown to the right.



Example: Marker Variable

Load the SAPI data.

```
dataDir <- "data"
sapi <- read.table(here::here(dataDir, "sapi.txt"),
                   header = TRUE,
                   na.strings = "-999")
```

Specify the **lavaan** model syntax for the SAPI extraversion CFA.

```
mod1 <- '
fun   =~ Q77 + Q84 + Q170 + Q196
liked =~ Q44 + Q63 + Q76  + Q98
'
```

Use the `cfa()` function to estimate the model.

```
library(lavaan)
out1 <- cfa(mod1, data = sapi)
```

Example: Marker Variable

```
partSummary(out1, 1:4)

lavaan 0.6-19 ended normally after 30 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        17

                                                  Used       Total
  Number of observations                           959        1000

Model Test User Model:

  Test statistic                               130.193
  Degrees of freedom                                19
  P-value (Chi-square)                           0.000
```

Example: Marker Variable

```
partSummary(out1, 5:7)

Parameter Estimates:

  Standard errors                              Standard
  Information                                  Expected
  Information saturated (h1) model           Structured

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  fun =~
    Q77               1.000
    Q84               0.761    0.051   14.902    0.000
    Q170              0.634    0.047   13.558    0.000
    Q196              0.795    0.046   17.381    0.000
  liked =~
    Q44               1.000
    Q63               1.512    0.147   10.278    0.000
    Q76               1.483    0.149    9.955    0.000
    Q98               1.243    0.119   10.462    0.000
```

Example: Marker Variable

```
partSummary(out1, 8:9)

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  fun ~~
    liked             0.231    0.025    9.234    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .Q77               0.548    0.038   14.389    0.000
   .Q84               0.727    0.039   18.703    0.000
   .Q170              0.687    0.035   19.572    0.000
   .Q196              0.364    0.025   14.731    0.000
   .Q44               0.662    0.034   19.291    0.000
   .Q63               0.807    0.048   16.943    0.000
   .Q76               0.966    0.054   17.931    0.000
   .Q98               0.469    0.029   16.121    0.000
    fun               0.627    0.056   11.303    0.000
    liked             0.182    0.029    6.290    0.000
```

Example: Fixed Factor

We only need to change one option to implement the fixed-factor method.

- The `std.lv = TRUE` option (i.e., standardized latent variables) applies the appropriate constraints.

```
out2 <- cfa(mod1, data = sapi, std.lv = TRUE)
```

Example: Fixed Factor

```
partSummary(out2, 1:4)

lavaan 0.6-19 ended normally after 17 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        17

                                                  Used       Total
  Number of observations                           959        1000

Model Test User Model:

  Test statistic                               130.193
  Degrees of freedom                                19
  P-value (Chi-square)                           0.000
```

Example: Fixed Factor

```
partSummary(out2, 5:7)

Parameter Estimates:

  Standard errors                                 Standard
  Information                                     Expected
  Information saturated (h1) model              Structured

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  fun =~
    Q77               0.792    0.035   22.606    0.000
    Q84               0.603    0.035   17.193    0.000
    Q170              0.502    0.033   15.180    0.000
    Q196              0.630    0.028   22.308    0.000
  liked =~
    Q44               0.426    0.034   12.580    0.000
    Q63               0.644    0.040   16.071    0.000
    Q76               0.632    0.043   14.845    0.000
    Q98               0.530    0.031   16.912    0.000
```

Example: Fixed Factor

```
partSummary(out2, 8:9)

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  fun ~~
    liked             0.683    0.033   20.483    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .Q77               0.548    0.038   14.389    0.000
   .Q84               0.727    0.039   18.703    0.000
   .Q170              0.687    0.035   19.572    0.000
   .Q196              0.364    0.025   14.731    0.000
   .Q44               0.662    0.034   19.291    0.000
   .Q63               0.807    0.048   16.943    0.000
   .Q76               0.966    0.054   17.931    0.000
   .Q98               0.469    0.029   16.121    0.000
    fun               1.000
    liked             1.000
```